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Dynamic Stability of V/STOL Aircraft at Low Speeds

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Various aspects of the longitudinal stability characteristics of propeller driven V/STOL aircraft at low speeds are described. Data on three tilt-wing configurations and a quad ducted propeller aircraft, from various experiments conducted with dynamic models, are presented and discussed. In the majority of flight conditions investigated, it was found that the measured transient motions could be satisfactorily approximated by linearized equations of motion. The stability derivatives determined from these experiments are presented, and the general trends of the stability derivatives in transition are examined. The linearized modes of motion typical of low-speed flight in transition are described. In certain of the low-speed flight conditions, nonlinearities are present, arising from the character of the static stability derivatives. These nonlinear motions are discussed. Stability derivatives from a single rotor and a tandem rotor helicopter are also presented for comparison purposes. It is noted that the helicopter has better stability characteristics in hovering flight than either the tilt-wing or quad duct vehicles. This is due to the fact that the helicopter has a smaller velocity stability and larger damping in pitch than the V/STOL vehicles of similar gross weight.

Nomenclature

c = mean aerodynamic chord, ft
g = acceleration due to gravity
I = moment of inertia in pitch
i_d = duct incidence ($i_d = 90°$ in hover)
i_T = thrust line incidence ($i_T = 90°$ in hover), equivalent to wing incidence or duct incidence
i_w = wing incidence ($i_w = 90°$ in hover)
L_0 = trim lift, lb
M = pitching moment, ft-lb, positive nose up
$M_u, M_w, M_q, M_{\dot{w}}, M_\alpha$ = stability derivatives, rate of change of pitching moment divided by moment of inertia with variable indicated in subscript
$M_\delta, M_\beta, M_{i_T}$ = control input terms, rate of change of pitching moment divided by inertia with control indicated in subscript
q = fuselage pitch rate ($\dot{\theta}$)
U = flight velocity, fps
U_0 = trim flight velocity, fps
u = horizontal perturbation velocity, stability axes, positive for forward motion of aircraft
w = vertical perturbation velocity, stability axes, positive for downward motion of aircraft
X = horizontal force, stability axes, positive forward
$X_u, X_w, X_q, Z_\alpha, Z_u, Z_w, Z_q$ = stability derivatives, rate of change of force divided by mass with variable indicated in subscript
$X_\delta, X_\beta, Z_{i_T}, Z_\delta, Z_\beta, Z_{i_T}$ = control input terms, rate of change of force divided by mass with control indicated in subscript
Z = vertical force, stability axes, positive downward
α = angle of attack ($\alpha \doteq w/U_0$), rad
α_{0L} = angle of zero lift, rad
β = propeller blade angle, rad
δ = longitudinal stick deflection
θ = fuselage pitch angle, positive nose up
σ = real part of characteristic root, per sec
ω = imaginary part of characteristic root, per sec
$(\dot{\ })$ = differentiation with respect to time

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Introduction

All types of V/STOL aircraft have encountered various stability and control problems at low speeds. It is the objective of this paper to discuss in over-all terms some features of the longitudinal dynamic characteristics of two types of V/STOL aircraft, the tilt-wing and the quad duct configuration.

It is important to note two aspects of the experimental determination of the stability derivatives of V/STOL aircraft that are often overlooked. First, is the fact that forces and moments due to the propellers, as well as those arising from the interaction of the propeller flowfield with other parts of the aircraft, are of significance. Therefore, it is important to conduct experiments to determine the horizontal velocity (or advance ratio) derivatives (X_u, Z_u, M_u). This is in contrast to a conventional subsonic aircraft, where it is usually assumed that the propeller contributions are negligible, and as a consequence, that the aerodynamic forces and moments acting on the aircraft vary as the square of the velocity.

Second, proper consideration should be given to the operating conditions of the propeller. It is necesssary that suitable constraints, corresponding to full-scale operation, be placed on the propeller so that the stability derivatives of the aircraft can be evaluated from the experimental data. For example, if the aircraft is designed to be flown with constant rpm using propeller pitch control, then wind-tunnel tests to determine stability derivatives should be conducted with these variables held constant. Performance studies on tilt-wing aircraft are often not conducted in this manner, making it difficult, if not impossible, to determine stability derivatives from the data presented.

The discussion in this paper will be restricted to the longitudinal stability in the speed range below 100 knots. Primary emphasis is given to the dynamic stability in trimmed level flight.

The majority of data presented has been determined from tests with dynamic models conducted at Princeton using the Princeton Dynamic Model Track and at NASA.[2-10] The instabilities of these aircraft as well as safety and other considerations involved in flight testing have made experimental data on the dynamic stability available from flight test very limited, except in the case of one early tilt-wing design where stability and control information was obtained.[11] The results of tests at Princeton[12] on a dynamic model of this aircraft agreed well with full-scale data where comparison was possible, giving a basis for confidence in the other results discussed.

The model tests have indicated, from direct transient measurements, that in many flight conditions, the dynamic motions of these aircraft can be approximated by conventional, linearized small perturbation equations of motion. The discussion that follows will be phrased primarily in these terms.

Nonlinearities were noted in two flight conditions during experiments with the tilt-wing model. These phenomena are discussed also.

Discussion

The linearized equations of motion used to describe the transient motions of an aircraft are

$$\dot{u} - X_u u - X_w w - X_q\dot{\theta} + g\theta = X_\beta \beta + X_{i_T} i_T$$

$$\dot{w} - Z_w w - Z_u u - (Z_q + U_0)\dot{\theta} = Z_\beta \beta + Z_{i_T} i_T$$

$$\ddot{\theta} - M_q\dot{\theta} - M_{\dot{w}}\dot{w} - M_w w - M_u u = M_\delta \delta + M_\beta \beta + M_{i_T} i_T$$

These are conventional small perturbation equations of motion, written for trimmed level flight, using stability axes.[1] For simplicity, we have assumed that the longitudinal stick position (δ) produces only a pitching moment. The tilting of the thrusting device (wing incidence or duct incidence) has been included as a control input on the right-hand side. The third control term is taken to be the propeller blade angle. If another means of controlling the propeller is used, then β can be taken to correspond to that control. Loosely speaking, β is the power control.

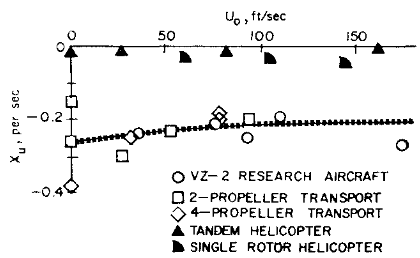

Fig. 1 X_u vs trim speed, tilt-wing aircraft and helicopters.

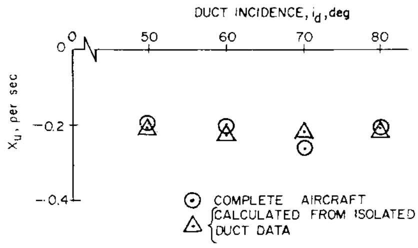

Fig. 2 X_u vs duct incidence, quad ducted propeller aircraft.

Dimensional derivatives are discussed. All stability derivatives presented have been scaled to represent aircraft of 40,000 lb gross weight.[3]

Helicopter derivatives are presented for a single rotor and a tandem rotor helicopter as determined from flight test.[13,14] These data were also scaled to a gross weight of 40,000 lb.

Stability Derivatives

Three of the stability derivatives in the equations (X_w, X_q, and Z_q) have been found to be generally unimportant[3,6] and will not be discussed further.

Longitudinal Force Derivatives

The rate of change of horizontal force with horizontal velocity (X_u), as determined by experiments on three tilt-wing aircraft of different configuration, is shown as a function of trim speed in Fig. 1.

Also shown for comparison are values for the two helicopters. It may be noted that for the tilt-wing vehicles, this derivative is large, and rather insensitive to configuration. The values for the quad duct aircraft are shown in Fig. 2. The value of X_u is of similar magnitude for both types of V/STOL aircraft. The helicopters exhibit a value that is roughly one-tenth of the value for these aircraft.

On the tilt-wing vehicle, the source of this large value is the fact that a horizontal speed change is summed with the slipstream velocity to produce an effective angle-of-attack change of the slipstream, when the wing incidence is high. The resultant force developed by the wing is essentially perpendicular to the slipstream velocity, and therefore a slipstream angle-of-attack change produces a large horizontal force. At the low forward speeds under consideration, the dominant dynamic pressure of the aircraft flowfield is produced by the slipstream. The ducted propeller vehicle exhibits a derivative of similar magnitude, due to the large momentum drag of the ducted propeller. This is shown in Fig. 2 where the value determined from isolated duct data is compared to the value measured for the complete aircraft.[15] No interference effects were included in the isolated duct calculation.

That a large value is inherent in various V/STOL aircraft may be seen by using the equations of motion to relate this derivative to the trim characteristics of the vehicle at low speeds. This stability derivative X_u is related to the thrust incidence, forward speed relationship in transition, and the relationship is particularly simple near hover. For a level flight, level attitude transition ($\theta = 0$, $w = 0$), near hover

$$X_u \approx g(di_T/du)$$

so that this derivative is directly related to the thrust incidence-horizontal velocity curve. Since i_T would correspond to shaft attitude on a helicopter, it may be seen that X_u will be perhaps ten times as large for a tilt-wing or tilt-duct vehicle as shown by the large differences in the shaft tilt-speed relationship.

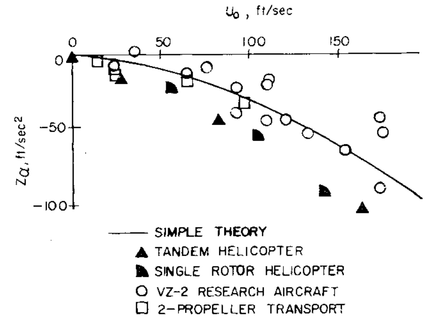

Fig. 3 $Z\alpha$ vs trim speed, tilt-wing aircraft and helicopters.

A second derivative of importance is Z_w, the rate of change of vertical force with vertical velocity. For forward speeds away from hover this derivative is related to the lift curve slope of the aircraft by the relationship $Z_\alpha = U_0 Z_w$. The variation of Z_α vs forward speed for the tilt-wing aircraft and the helicopters is shown in Fig. 3.

This derivative can be approximated by

$$Z_\alpha \approx -g/\tan(i_T - \alpha_{0L})$$

This crude approximation agrees quite well with experiment as shown in Fig. 3. If stalling is present, then the derivative will be smaller in magnitude than the value given by this approximation. Suitable experiments have not been conducted to provide data on the value of Z_w in hover, a derivative that is of importance in the vertical control of the aircraft.

The remaining force derivative, the rate of change of vertical force with horizontal velocity (Z_u) is somewhat complex to generalize as it is more sensitive to the details of the trim conditions. Theoretical calculations[16] indicate that Z_u is quite sensitive to the wing incidence-speed relationship. The variation of Z_u in this speed range is shown in Fig. 4.

There is considerably more variation in values of this derivative among the three tilt-wing configurations considered; the helicopter data show similar values at low speeds. Symmetry considerations yield a linearized value of zero for Z_u in hover.

The force derivatives Z_α and Z_u for the quad configuration ducted propeller aircraft are shown in Fig. 5.

The approximation given previously for Z_α agrees quite well for the quad data. Also shown, for comparison purposes, are the derivatives determined from isolated duct data, assuming no interference effects between the forward and aft ducts. The good agreement between the derivatives of the complete vehicle and the isolated duct data indicates that the force derivatives are insensitive to the configuration layout.

Pitching Moment Derivatives

The damping in pitch (M_q) has been found to be small for the two V/STOL aircraft for which data are shown in Fig. 6.

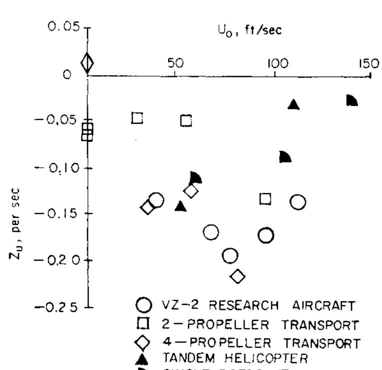

Fig. 4 Z_u vs trim speed, tilt-wing aircraft and helicopters.

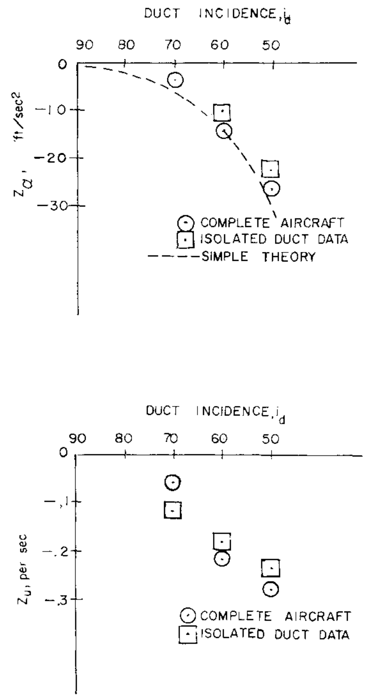

Fig. 5 Z_u, Z_α vs duct incidence, quad ducted propeller aircraft.

There is no large contribution from any of the components of the aircraft until the forward speed becomes large enough to provide a significant dynamic pressure at the horizontal tail or other rear lifting surface. Note the considerably larger values for a single and tandem rotor helicopter.

There is no experimental information on the derivative $M_{\dot{w}}$. Most of the experimental techniques used to evaluate the pitch damping determine the combination $(M_q + U_0 M_{\dot{w}})$.[2,5] Indications are that $M_{\dot{w}}$ is small; however, this has not been adequately verified. The formulation of this derivative for conventional aircraft, assumed to arise from a lag in a downwash change at the wing reaching the tail, raises questions as to the analytical treatment of this effect, as well as its importance, at low speeds. Insufficient data is available at the present time to comment further.

Since the derivatives examined so far, X_u, Z_w, and M_q, are similar for the vehicles, differences in the dynamics will arise from the two remaining static stability derivatives M_u and M_w.

The rate of change of pitching moment with vertical velocity (M_w), which at forward speed is directly related to the angle-of-attack stability, $M_\alpha = U_0 M_w$, follows quite a general trend in many V/STOL vehicles, as pointed out some years ago.[17] That is, at low speeds the derivative is positive (unstable) and tends to change sign as forward speed in-

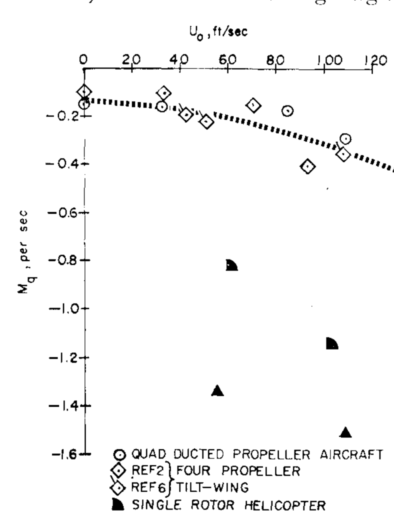

Fig. 6 M_q vs trim speed.

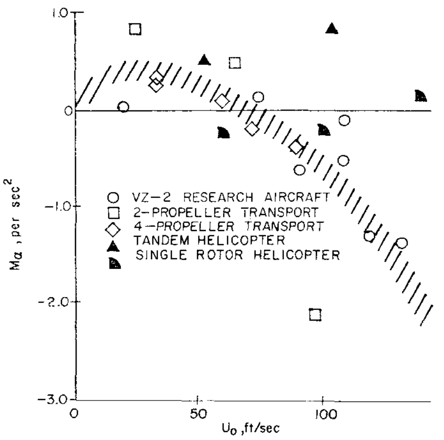

Fig. 7 M_α **vs trim speed, tilt-wing aircraft and helicopters.**

creases as shown in Fig. 7. The helicopters generally show the opposite trend.

Theoretical calculations[16] in the wing incidence range where the linearized derivative is near zero are shown in Fig. 8. A significant nonlinearity may be noted even though these calculations are based on a linear variation of wing lift with angle of attack.

It is common on tilt-wing aircraft to have two controls to provide pitching moments. The proportion of the pitching moment provided by each of the controls may affect the stability derivatives. In comparing the results of various tests[2,6] the effect of two extreme settings of horizontal tail incidence may be noted. From the curves shown in Fig. 9, a low-incidence setting used in one series of tests[2] resulted in stall at negative angle of attack and a high tail incidence employed in other experiments[6] caused the tail surface to stall a positive angle of attack, and as a result quite different pitching moment curves for the same aircraft occur in similar flight conditions.

The data from the quad duct model presented in Fig. 10 show a trend for the variation of the derivative M_w in the low-speed portion of transition similar to that of tilt-wing aircraft. Shown on the graph is the value of M_w calculated using isolated duct data, with no interference between ducts.[15] The increment calculated, including the aft lifting surface which is the only stabilizing contribution, is also shown. It has been assumed that the fuselage contribution is not important. The difference between the isolated contributions and the complete aircraft is undoubtedly due to the fact that interference effects have been neglected. However, evaluation of interference contributions is not possible with the limited data available on flowfields around ducted propellers in this speed range.

The velocity stability (M_u) is large and positive at low speeds for the various tilt-wing aircraft as shown in Fig. 11.

Theory[16] was quite successful in predicting values near hover, but was not successful at increased forward speed.

The stability derivative M_u is very sensitive to the manner in which the aircraft is trimmed. For example, on a tilt-wing aircraft with a tail rotor and a horizontal tail, the value of this derivative depends strongly on the relative proportion of pitching moment provided by the tail rotor and the horizontal tail.

It is difficult to generalize the nature of this derivative through transition, as seen by the variation among the three tilt-wing configurations, except for the trend of a fairly marked decrease with increasing speed away from hover.

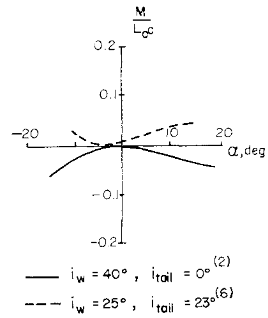

Fig. 9 Pitching moment vs angle of attack. Tilt-wing aircraft (experiment).

Static Stability

Static stability can be conveniently related to the terms in the equations of motion presented earlier. The characteristic equation of the vehicle is of fourth order and the static stability of the vehicle is defined by the sign of the constant term in the characteristic equation. The sign of this term will normally indicate whether one of the modes of motion of the aircraft is a pure divergence or not. This term is equal to

$$-gZ_w(M_u - M_w Z_u/Z_w)$$

and is positive for static stability.

The term in parentheses can be identified as the total pitching moment change with speed when lift equals weight (at constant propeller pitch angle and wing incidence). The phrase "constant power" is sometimes used to describe this constraint. Precisely, to determine this quantity from flight test, the deflection of the lever controlling the propeller thrust should be held constant while the flight speed is varied. This quantity in parentheses is directly related to the stick position variation with speed by

$$(d\delta/du)|_{\beta, i_T} = -(1/M_\delta)(M_u - M_w Z_u/Z_w)$$

There are a number of other stick position gradients of interest in a variable configuration aircraft. In a level flight transition, for example, where wing incidence and propeller blade angle are varied to keep the aircraft in level flight at zero fuselage attitude, the longitudinal stick position with air speed gradient can be expressed as

$$\left.\frac{d\delta}{du}\right|_{\substack{\theta=0 \\ w=0}} = -\frac{1}{M_\delta}\left(M_u + M_\beta \frac{d\beta}{du} + M_{i_T}\frac{di_T}{du}\right)$$

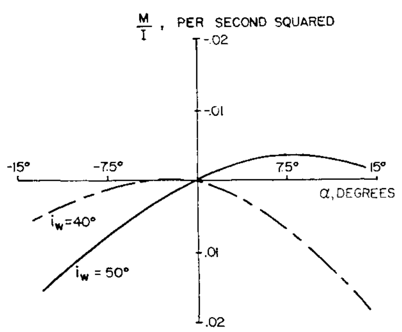

Fig. 8 Pitching moment vs angle of attack for various wing incidences. Tilt-wing aircraft (theory).

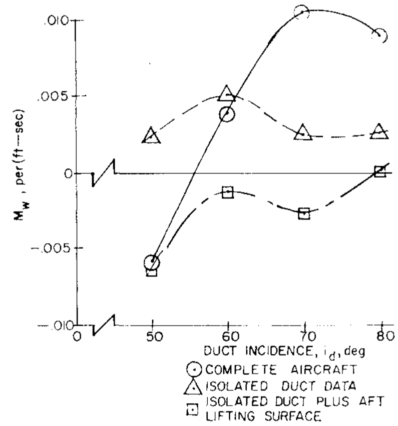

Fig. 10 M_w **vs duct incidence, quad ducted propeller aircraft.**

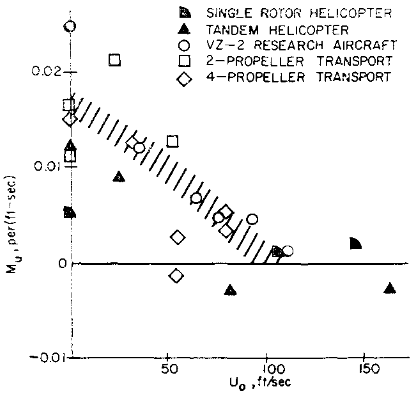

Fig. 11 M_u vs trim speed, tilt-wing aircraft and helicopters.

The coefficient of the last term of this expression, the rate of change of pitching moment with thrust incidence, will usually be large and will cause the variation of stick position with speed in level flight to be quite different from the variation related to the static stability characteristics. Comparison of these slopes for a tilt-wing aircraft[11] is shown in Fig. 12.

Various other gradients are also of interest, and these can be developed readily from the equations of motion. This viewpoint tends to directly relate the static and dynamic stability characteristics of a vehicle.

Modes of Motion

Now then, let us consider briefly the dynamic stability characteristics that result from the stability derivatives as described. The modes of motion of a tilt-wing aircraft during transition are not identifiable in terms of the conventional phugoid and short period motions. In particular, the convenient separation into a slow motion and a fast motion may not occur. We may see why this separation will not normally be present at low speeds, for the aircraft described, by considering first the dynamics of a vehicle with M_u and M_w equal to zero. In this case (with X_w neglected), the characteristic equation will consist of a series of perfect factors. The roots corresponding to the modes of motion will be X_u, Z_w, M_q, 0. Each of these modes can be identified with a particular degree of freedom; Z_w with vertical velocity, X_u with horizontal velocity, and M_q and 0 with pitch angle. Now, it is typical of conventional aircraft that $|M_q|$ and $|Z_w|$ are considerably larger than $|X_u|$ indicating that the faster motions of an aircraft tend to occur in vertical velocity and pitching velocity compared to horizontal velocity. However, at low speeds, for the type of aircraft under consideration, these three derivatives are all of similar size, and a separation of time scales is not present, as shown in Fig. 13.

To gain some insight into the modes of motion, consider the variation of the characteristic roots as M_w is increased in a stable sense. It is seen from Fig. 13 that a short period and a phugoid develop, and are well separated and rather loosely coupled in the case where M_q and Z_w are well separated from X_u. If this is contrasted with a typical low-speed case, also shown, where all of the poles of the locus (X_u,M_q,Z_w) are of

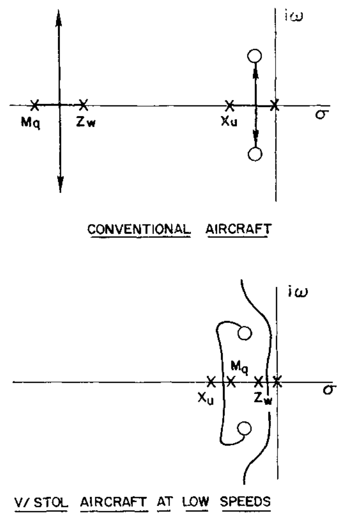

Fig. 13 Root locus for M_w increasing in stable sense (M_u < O).

similar size, a complicated and sensitive locus results. It is difficult to generalize the character of the modes of motion that may be present in the low-speed portions of transition.

Now, consider the end points of transition for the tilt-wing aircraft. In aircraft flight, the modes of motion consist of a phugoid and short period oscillation. In hovering, the vertical velocity is uncoupled, or weakly coupled to the pitch angle and the horizontal velocity because of the symmetry of the trim condition. Pitch angle and horizontal velocity are strongly coupled by the large positive velocity stability, resulting in an unstable oscillation. This large velocity stability produces a relatively fast unstable motion with a period of the order of 10 sec. In addition to this unstable oscillation, there are two convergent modes, one corresponding to the vertical motion and one associated with the pitch angle and horizontal velocity.

As the forward speed is increased, the vertical motion becomes coupled to the pitching and translational degrees of freedom. There will be a smooth variation of the characteristic roots from hover to aircraft flight. Two possibilities for this variation are shown in Fig. 14.

One variation is shown for a comparatively large value of Z_w and the other for Z_w equal to a small value. For a large Z_w, the hovering roots decrease in frequency as speed increases, and become stable, and may be identified as the phugoid in aircraft flight. For the small value of Z_w, the hovering oscillation increases in frequency as the trim speed is increased, and may be identified as the short period in aircraft flight. Similar trends will be noted for large and small values of M_q.

Thus, the general nature of the instabilities encountered in low-speed flight will be as follows. For hover and slow forward speeds, the large velocity stability produces an oscillatory instability of comparatively short period with a rapid rate of growth due to the low angular damping. At higher speeds, due to the decrease in the velocity stability this oscillatory motion will tend to become stable. A purely divergent motion may occur in this flight regime due to static instability of the aircraft. The divergence occurs when M_u decreases to a small value while M_w is still positive (unstable).

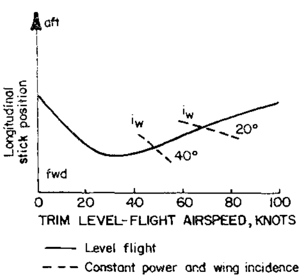

Fig. 12 Stick position vs airspeed, tilt-wing aircraft (flight test).

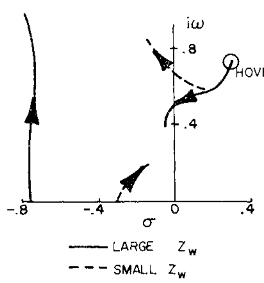

Fig. 14 Locus of roots through transition showing sensitivity to Z_w. Tilt-wing aircraft.

As speed is further increased, the stable trend in the angle-of-attack stability as well as the generally increasing damping in pitch will result in a stable aircraft. Similar trends of the modes of motion occur on the quad configuration.

In hovering and at very slow speeds, the two helicopters possess considerably better dynamic stability characteristics owing to the much larger damping in pitch, and the much smaller velocity stability. In the intermediate speed range considered here, the dynamics of the two helicopters would be quite similar to the tilt-wing aircraft.

Nonlinear Phenomena

Two particular occurrences of nonlinear phenomena were noted from experimental measurements of the transient response of a dynamic model of a tilt-wing aircraft, one associated with nonlinearities in M_u and the other associated with M_w.[2]

As shown in Fig. 11, there is a flight regime where M_u decreases rapidly with decreased incidence and correspondingly increased speed. Consider a response initiated at a trim condition in the middle of the rapidly decreasing portion of this curve, a nose up disturbance with a corresponding decrease in speed, will bring the aircraft into the region of larger M_u and result in an oscillatory instability. A nose down disturbance causes an increased speed bringing the aircraft into a region where it is statically unstable, with a resulting divergence. The difference in these two responses is shown in Fig. 15.

At lower wing incidences, in the flight condition where the linearized value of M_α is small, nonlinearities in M_α caused the response characteristics to differ also, depending upon the direction of the disturbance. The nonlinear behavior is particularly noticeable because both of the static stability derivatives are small, and M_α depends both on angle of attack and on speed. Two responses in different initial directions are shown in Fig. 16. The theoretical and experimental variations of pitching moment with angle of attack in this flight condition are shown in Figs. 8 and 9.

An initial nose up disturbance results in a relatively small amplitude oscillation, arising from the stable slope of the pitching moment vs angle of attack shown in Fig. 9. The small amplitude nose down motion is divergent, due to the unstable slope of pitching moment vs angle of attack. However, the large increase in speed that arises from the nose down motion, places the aircraft in a region where the slope of pitching moment vs angle of attack is stable for both positive and negative incidence. The scalloped nature of the curve indicates, however, that the slope is considerably steeper for positive angle of attack than for negative angle of attack.

Generally, one would expect nonlinearities to be of importance in regions of flight where both of the linearized static stability derivatives are small, and it is difficult to characterize all the various phenomena that may occur.

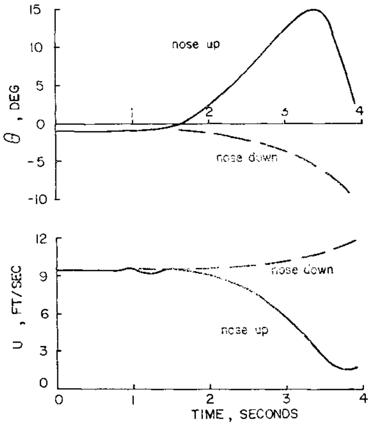

Fig. 15 Transient response in region of rapidly changing M_u. Tilt-wing aircraft (experiment).

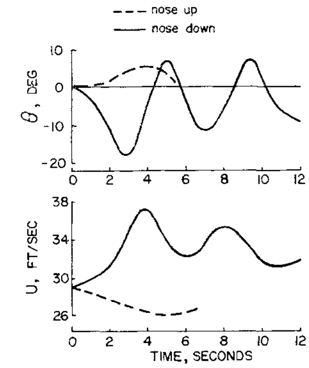

Fig. 16 Transient response, nonlinear M_α, tilt-wing aircraft (i_w = 40°, i_{tail} = 0) (experiment).

The quad configuration transient motions did not exhibit any noticeable nonlinearities in this flight regime although similar trends in the derivatives M_u and M_w with speed have been noted.

Concluding Remarks

We have discussed several aspects of the longitudinal dynamic stability characteristics of propeller driven V/STOL aircraft at low speeds. Transient response measurements with dynamic models have indicated that a conventional linearized description applies in many level flight trim conditions.

The aircraft under consideration, tilt-wing and quad duct V/STOLs exhibit a number of similarities. In hovering flight, they are characterized by low angular damping and high velocity stability. The size of these derivatives is such that the transient motion will be faster than that of a helicopter of similar size.

The general trend of some of the stability derivatives may be estimated, however other important derivatives are dependent upon configuration details. The velocity stability derivative M_u decreases as thrust incidence is reduced and speed increases from hover. Flight conditions are encountered in the low-speed portion of transition where nonlinearities may be of significance. The velocity stability and the angle of attack stability are likely to be near zero.

A detailed definition of the flowfield in the neighborhood of wing-propeller and propeller-duct combinations is necessary in order to predict interference effects including the downwash at horizontal tail locations.

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Fluctuating Flowfield of Propellers in Cruise and Static Operation

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The flowfields computed in the lifting-line approximation for finite-bladed, lightly loaded propellers in axial cruise are compared for constant and distributed blade circulations of the same total loading. Regions of agreement and differences in the two flowfields are distinguished for various blade numbers and advance ratios, and the asymptotic behavior of the velocity components is established. Related computations and comparisons are made for heavily loaded propellers in static operation, i.e., hover. A propeller in static operation is characterized by a greatly distorted trailing vortex system and the appreciable radial contraction and axial extension involved are modeled in the present numerical computations. The azimuthal variations of the velocity components in the propeller plane are presented for three cases which allow the velocity variations to be examined for three- and four-bladed propellers with similar effective pitch of their trailing vortex systems, and for two three-bladed propellers with widely different effective pitch. The relationship between the mean inflow at a given radius in the propeller disk plane and the instantaneous inflow to the blades at the same radius is examined for these same three cases.

Nomenclature

C_T = propeller thrust coefficient, $T/\frac{1}{2}\rho U^2\pi R^2$
$c_m{}^u$ = Fourier harmonic coefficient of u
J = advance ratio, $U/\Omega R$
m = harmonic number
N = number of propeller blades
R = propeller tip radius
T = propeller thrust
U = axial flight velocity
u,v,w = axial, radial, and tangential induced velocity components, respectively
x,r,θ = cylindrical, propeller-fixed coordinate system
Γ = propeller blade circulation
ρ = fluid density
Ω = propeller angular velocity
$\langle\ \rangle$ = azimuthal mean of induced velocity

Subscripts

p = value at propeller blade
0 = value in propeller plane

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I. Introduction

FOR a finite-bladed, lightly loaded propeller in axial cruise flight, the harmonics of the induced velocity components were computed numerically at several field positions in Ref. 1. The propeller was represented by a lifting line and was assumed to have a constant blade-circulation distribution.